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# GERALD J. LIEBERMAN

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*To Jerry Lieberman, our student, teacher, colleague and most important, our friend, on the occasion of his 70th birthday*

## EARLY DAYS

Gerald J. Lieberman was born on December 31, 1925, in Brooklyn, New York, after a hectic New Year's Eve trip to the hospital. His father, Joseph, spelled his last name Liberman, but his mother, Ida, preferred Lieberman, the spelling that she and some of Joseph's siblings used. Joseph and Ida had come to this country from Lithuania. Joseph worked for the Metropolitan Life Insurance Company, and they lived in an "historic" section of Flatbush. The much wanted baby boy was the center of the family, which included two doting older sisters,

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Jerry, circa 1960.

Shirley and Rosalind. He grew fast—one of the tallest boys in nearby Public School 197—and achieved his adult height at about the age of 13. As a boy, he was described as towheaded and gawky. Jerry did not realize that he had a middle initial until he was 15 and needed a birth certificate to get a work permit. Jerry asked his parents if they had given him a middle initial, but they did not remember. In any case, since the J does not stand for anything, Jerry likes to quip that his middle name is *Jinitial*.

At James Madison High School he was an honor student. When asked if his brightness showed in his youth, one of his sisters hesitated and then said, "No other alternative was considered." Fortunately he could live up to expectations. One of his classmates—still a close friend—said he scored academically in the high 90s when such numbers meant something. No devotee of promptness, at high school he and his two best pals were designated Co-Captains of the Late Team. It would have been hard to predict that a remarkably effective, efficient, and punctual administrator would develop from this beginning.

In his youth, there were a few clues to his later lifestyle. He sold Good Humors at Brighton Beach—a precursor of many trips to local ice cream parlors (even after a Chinese dinner) and at home a freezer well stocked with goodies in case a "craving for sweets" developed at a random moment. For some reason the freezer is kept locked today. Fondness for delicatessen food also began in Brooklyn.

Perhaps the only disappointment to a family of music lovers is that Jerry was found to be tone deaf.

But the characteristic that was earliest evident was his warmth, generosity of spirit, and affection for family and friends. Indeed, many of his early friends became part of an extended family into which many of us have been fortunate enough to be adopted.

Surviving a highly competitive admissions process, he was accepted into Cooper Union and graduated in Mechanical Engineering in 1948. He took his master's degree in Statistics from Columbia in 1949. On graduation he went to work under Churchill Eisenhart at the National Bureau of Statistics (NBS).

## COMING TO STANFORD

Meanwhile, back at Stanford, Allen Wallis had returned to the Economics Department from wartime service as Director of the Statistical Research Group (SRG) at Columbia. He was approached by the Office of Naval Research (ONR) to negotiate a contract to continue some of the SRG's work in acceptance sampling with the additional agenda of encouraging the development of statistics at Stanford. Wallis left for Chicago but suggested that Albert Bowker be hired to carry on the contract, later administered by Mina Rees, and to help build a Statistics Department. Bowker had come to Stanford but also did some consulting work for Eisenhart at the NBS, where he worked with Jerry and recruited him as a graduate student in 1950. (Needless to say, Wallis, Rees, Eisenhart, Bowker, and Girshick had all worked together in wartime research work.) Jerry took his Ph.D. in Statistics in 1953 with Bowker and Girshick.

Housing for graduate students was scarce and expensive in the 1950s. The Statistics Department was able to place Jerry in modest accommodations being vacated by another graduate student, Lincoln Moses, on Perry Lane. The new residence was a combination campsite and cottage with heat from a fireplace and open cracks in the walls encouraging wildlife. It did have indoor plumbing. The rent was $23 a month—and worth every penny. Our Brooklyn boy was terrified but made a sensible decision not to go it alone. He sent for Helen, who had grown up in Sioux Falls, South Dakota, and worked with Jerry at the NBS; married her; and in a few months moved to the more civilized realm of College

Terrace—a move made more feasible by Helen joining the computing staff of the new department and pounding a Monroe calculator all day.

Jerry and his family are very close. He and Helen are justifiably proud of their four grown children, Diana, Janet, Joanne, and Michael, and of their two grandchildren. If you visit the Liebermans, you will invariably find one or more of the children there. His sister, Shirley, regularly flies in to see him from New York. Jerry has a nice sense of humor and encourages his children to join in on the fun when guests come to visit. For example, if you came to visit him when Janet was 3, Jerry would ask Janet in front of you, "What is the integral of $2x\,dx$ from zero to one?" Without a moment's hesitation, she would astonish you by blurting out with a grin, "One!" By the time you had begun to wonder how a child so young could be so precocious, she had run off laughing and Jerry was on to another subject, making you feel foolish for doubting them.

Jerry loves gadgets. He is invariably the first of his friends to obtain the latest and best TV, VCR, camcorder, and so on. He is also handy around the house, as, for example, with plumbing, electrical work, and the like. In fact, his friends often call him to rescue them from one disaster or another such as clogged plumbing, making complex TV/VCR connections, and so forth. His friends also call him to find out the best place to buy things because he is a storehouse of knowledge on such matters.

## DEPARTMENT OF STATISTICS

One of the themes in the formation of the Department of Statistics in 1948 was joint appointments between Statistics and other departments. In keeping with this principle, Jerry joined Stanford's Departments of Industrial Engineering and Statistics in 1953, thereby providing a link with the School of Engineering. He rose from Assistant Professor to Professor in 6 years.

The Department of Statistics built up rapidly. Between 1953 and the mid-1960s, the senior faculty at different times consisted of Theodore Anderson, Kenneth Arrow, David Blackwell, Albert Bowker, Herman Chernoff, Kai Lai Chung, Abe Girshick, Vernon Johns, Samuel Karlin, Rupert Miller, Lincoln Moses, Ingram Olkin, Emanuel Parzen, Herbert Scarf, Herbert Solomon, Charles Stein, Patrick Suppes, and Harvey Wagner. Jerry was instrumental in the development of curricula for the master's and doctoral programs. In particular, he developed and taught statistics courses for engineers, which were very popular.

## DEPARTMENT OF OPERATIONS RESEARCH

Operations research emerged during World War II as a new discipline that developed and used mathematical models for decision making. Stimulated by the development of the computer, the field developed rapidly after the war spreading into industry and other areas of government. Programs in operations research were started at several universities in the mid-1950s. At Stanford, interest in operations research developed first in the mid-1950s led by Kenneth Arrow, Samuel Karlin, Herbert Scarf, Harvey Wagner, and Jerry. By the latter half of the 1950s, the Departments of Industrial Engineering and Statistics were offering courses in the field.

Because of the importance of this new discipline and the significant research and teaching activities in the area that had developed in several departments and schools by the late 1950s, Jerry and Kenneth Arrow suggested to Albert Bowker, Dean of Graduate Studies, that he form a committee in the early 1960s to explore the possibility of coordinating these activities. Bowker did so and appointed Arrow to chair that committee. The committee recommended that an interdepartmental committee be established to administer a Ph.D. program in Operations Research. The university approved the proposal and the committee was formed in 1962 with Jerry as its first chair. The Committee in Charge also included Kenneth Arrow, James Howell, Samuel Karlin, Alan Manne, Herbert Scarf, Daniel Teichrow, and Harvey Wagner. Charles Bonini, Frederick Hillier, Roy Murphy, Arthur Veinott, and Peter Winters joined the committee soon thereafter, with Ronald Howard and Robert Wilson following a bit later. The program was an immediate success with seven students taking Ph.D. degrees in 1965. That year, Jerry convinced Stanford to provide a new billet to the program that was filled by George Dantzig in 1966.

In the fall of 1966, Arthur Veinott proposed to Joseph Pettit, Dean of the School of Engineering, that a Department of Operations Research be established in the school. Pettit said that he would support the idea if the rest of the operations research faculty felt the same way. They did, and Jerry negotiated with Pettit to establish the Department of Operations Research in the School of Engineering in 1967. Jerry played a central role in developing these arrangements, successfully secured billets to bring Richard Cottle and Donald Iglehart to Stanford, and was appointed the department's first Executive Head, a position which was retitled 'Chairman' in 1969 and which he held until 1975. Besides Jerry, the department included Kenneth Arrow, Richard Cottle, George Dantzig, Frederick Hillier, Rudolf Kalman, Donald Iglehart, Alan Manne, and Arthur Veinott. Curtis Eaves joined the faculty in 1970. Nine of these ten remain in the department to this day.

The Program in and Department of Operations Research, like other new activities, was underbudgeted by the university. Office space and student support were short. Jerry worked hard to secure the needed resources from the university and elsewhere. In the early 1960s, he got the university to refinish an old house for the program with space for faculty, but not students, and then to provide space for students in remote nooks and crannies of the campus. He kept pushing for better facilities and in the late 1960s obtained a much nicer air-conditioned and remodeled section of an old campus building for the department with space for both faculty and students. He also tapped his network of friends in local industry to help support students by providing part-time work for them.

Jerry drew on his experiences in the Department of Statistics in providing leadership to the new department. One of his first initiatives was to successfully advocate supplementing the Ph.D. degree with an M.S. degree program. Research grants and contracts were more readily available in those days, and there was some initial skepticism about his proposal because of the worry that the M.S. program would interfere with the Ph.D. program. Jerry allayed these concerns by arguing that it was important to provide professional education to a larger group of students that would have a greater impact in industry and government and also provide a solid enrollment base for this graduate department. His wisdom on this point was prophetic when grants and contracts became more difficult to secure in the 1980s and 1990s in the face of rising demands for them and declining constant-dollar support.

Jerry fostered an unusually cordial environment in the department that has continued for three decades. He did this in several ways. Except on days when he had other commitments, Jerry would collect the faculty for lunch to drive to local off-campus favorites of those days such as Bib-and-Tuck, Kirks, Stickneys, and Harry's Hoffbrau. True to his Brooklyn heritage, Jerry especially liked pastrami and roast-beef sandwiches. These lunches were almost like mini daily department meetings where ideas about the department could be proposed and discussed.

Jerry would often visit faculty members in their offices to chat individually and keep in touch with any problems that might be arising. He had a knack for keeping the faculty well informed about things they needed to know while at the same time never violating a confidence. He was a good listener and was always sympathetic to problems of faculty, staff, and students. He would give good advice and help where this was possible. Jerry respected and accommodated individuality, for example, by allowing faculty members to furnish their offices as they pleased. Faculty members who went on leave could be secure that Jerry would consult with them on departmental matters and defer decisions to which they objected until their return. If a faculty member wanted to teach a course, bring a visitor, or have summer support, Jerry always tried to make this happen. When support was required, Jerry often provided it from his own research grants, especially his long-standing ONR contract.

Jerry believed strongly that the department should be run democratically and that resources such as office space should be allocated equally. This helped to create a happy environment of fairness in the department. But it did embarrass him on one occasion when he had the faculty draw straws for offices and a particularly senior and eminent member of the department ended up with the shortest straw and smallest office. To Jerry's relief, one of the younger faculty solved the problem by swapping the large office to which he was entitled for the small one.

Jerry felt that having outstanding faculty was essential to a successful department and for this reason supported excellence in appointments. But he also thought important decisions such as appointments and major curricular changes should be made unanimously because he felt that doing otherwise is divisive. He did not shrink from twisting arms to achieve consensus. However, on one occasion in which he dearly wanted to appoint an individual to the faculty and could not achieve unanimous agreement, he stuck to his principles and did not try to appoint the individual though he probably could have done so.

## STUDENTS AND TEXTBOOKS

Jerry enjoys teaching students at all levels. He is a well-organized and popular teacher who gives freely of his time to help students learn. He gives nice problems to his Ph.D. students and carefully monitors their progress to help them be productive and avoid blind alleys. He makes sure that they complete their degrees in a timely manner and helps them find positions to their liking. For these reasons, Jerry is a popular thesis advisor. To date, Jerry has been the Principal Advisor of 20 Ph.D. students in Operations Research, 13 in Statistics, and 2 in Industrial Engineering. His students populate many leading universities, businesses, and research laboratories and are listed here:

| | | |
|---|---|---|
| S. Christian Albright, Jr. | Yukio Hatoyama | Joseph J. Schoderbek |
| Lloyd F. Bell | Frederick S. Hillier | Robert S. Shipley |
| Frederick M. Biedenweg | Joseph H. Kullback | Andrew W. Shogan |
| David A. Butler | Thomas P. McWilliams | Matthew Sobel |
| Timothy M. Corcoran | Frederick A. Miercourt | Leland T. Stewart |
| David C. Dellinger | Elias A. Parent, Jr. | Howard M. Taylor III |
| Eugene P. Durbin | Mark M. Perkins | John V. Wagner, Jr. |
| Randall E. Fleming | William P. Pierskalla | Kenneth T. Wallenius |
| Ronald E. Glaser | Marion R. Reynolds, Jr. | Alan P. Wood |
| Lola P. C. Goheen | Donald B. Rosenfield | W. Max Woods |
| Gary B. Gottlieb | Sheldon M. Ross | Peter W. Zehna |
| Geoffrey Gregory | Tapas Sarkar | |

Jerry is a superb expositor. He has the ability to merge the practical with the theoretical in a manner that enhances each. He exhibited this talent by writing innovative basic textbooks. His first two books were written with Albert Bowker. These books focus on statistical methods as they are used in engineering. They have been used for nearly 40 years and served as the training ground for several generations of engineers.

The book by Bowker and Lieberman, *Engineering Statistics* [11] (an elaboration of their earlier 1955 *Handbook of Industrial Statistics* [8]), first published in 1959, was developed as the text for Jerry's engineering statistics course and has served as the definitive reference and text for several decades. Although other statistics books for engineers appeared later, the one by Bowker and Lieberman remained as a cornerstone—it was the gold standard used to calibrate its successors.

One of Jerry's undergraduate advisees in Industrial Engineering whom Jerry interested in operations research was Frederick Hillier. Hillier took his Ph.D.

with Jerry in Industrial Engineering in 1961. After visiting eastern universities on a recruiting trip and telling his wife to expect to move east, Hillier, marooned in New York by its worst blizzard in a decade, accepted shelter from Jerry, on leave at Columbia. Jerry took advantage of this to persuade Hillier that life on the West Coast was more hospitable, and Hillier joined the Stanford faculty. In the early 1960s, he and Jerry noticed that there was a need for an up-to-date text in operations research for undergraduates. They responded by writing one of the most widely used textbooks in this field, *Introduction to Operations Research* [12], with several hundred thousand copies in circulation and now in its 6th edition. The impact of this text in educating a wide range of students in the elements of operations research has been profound. Many introductory texts have been written in this field, but their book stands out for its longevity and its wide appeal, especially to engineers. Their text has revealed the value of the field and introduced some of its enduring ideas to a vast number of young people—interesting a not inconsiderable number in graduate work in operations research. For a more complete discussion of this book, see Hillier [4].

## SENIOR LEADERSHIP AT STANFORD UNIVERSITY

While Jerry was Chairman of Operations Research, he used to joke that he would never become a dean. How wrong he was! In 1975, Halsey Royden, a mathematician and old friend who was then Dean of the School of Humanities and Sciences, asked Jerry to help him by serving as Associate Dean. Jerry obliged and began a decade of service in the senior administration at Stanford including 2 years as Associate Dean, 3 years as Vice-Provost and Dean of Research (including a short stint as Acting Vice-President and Provost), and 5 years as Vice-Provost and Dean of Graduate Studies. Among his important accomplishments during this period was his major role in arranging a budget allocation to the Engineering School that significantly reduced charges of academic-year faculty salaries to research grants and contracts, thereby reducing the dependence of that school on government support. This move was very popular with the faculty and helped to mitigate the effects of subsequent reductions in government support for basic research. Also, he led the effort to bring Kenneth Arrow back to Stanford in the late 1970s after a decade at Harvard. Jerry returned full time to the Department of Operations Research in 1985.

Jerry continued his university leadership mainly by participation in the Academic Senate, including a year as Chair, and on the Advisory Board until 1992 when he accepted the request of Stanford President Donald Kennedy to join him in the administration as Provost. When Kennedy was succeeded as President by Gerhard Casper in the autumn of 1992, Jerry stayed on another year as Provost until Casper chose a new Provost in the summer of 1993. Jerry helped to maintain stability in the university during this difficult period. He became Professor Emeritus on September 1, 1994.

## PUBLIC SERVICE AND LEADERSHIP OF NATIONAL SOCIETIES

Jerry has a broad record of national leadership in statistics, quality control, and operations research. As a brief indication of this, the national offices he has held in four professional societies in these fields are listed here:

American Statistical Association (Chairman, Section on Physical and Engineering Sciences, 1961 and 1973; Vice-President, 1963-64; Member of Board of Directors, 1974-76)

Institute of Mathematical Statistics (National Treasurer, 1960-64; Member of Council, 1967-70)

American Society for Quality Control (National Director, 1958-63)

The Institute of Management Sciences (Member of Council, 1973-75; President-Elect, 1979-80; President, 1980-81)

In addition, Jerry has served on the editorial boards of *Industrial Quality Control* (1960-65), *OMEGA (International Journal of Management Science)* (1973-present), and the *Naval Research Logistics Quarterly* (1977-86). As an expression of the high regard in which he is held by his peers, the Joint Councils of the Operations Research Society of America and of the Institute of Management Sciences on October 31, 1993, unanimously resolved the following:

> For his 40 years of dedicated service to the profession of operations research and management science, the Councils of ORSA and TIMS recognize our dear friend and colleague Gerald J. Lieberman. Jerry was President of TIMS in 1980-81 and has been an influential member of many key committees. As the founder of the Stanford Operations Research Department, he has helped develop many of the leaders of ORSA and TIMS. What we treasure most, however, is the mentoring and kindness that Jerry has bestowed on literally hundreds of his students and colleagues. An encouraging word, a pat-on-the-back, a helpful phone call, radiating enthusiasm over other's achievements—all of these are daily activities for Jerry. Our profession is enriched and our sense of community stronger because of the continuing role model that he has set. We honor one of our most active and accomplished colleagues, Jerry Lieberman.

Jerry's advice is often sought for advisory panels and boards. His extensive public service in these areas follows:

Advisory Panel for Mathematical Sciences, National Science Foundation (1968-73)

Maritime Transportation Research Board of National Research Council (1966-71)

Board of Advisors, Naval Postgraduate School (1976-85)

Panel on Applied Mathematics Research Alternatives for the Navy of the National Research Council (1977-89)

Committee on Applied and Theoretical Statistics of the National Research Council (1978–81)

Panel for Applied Mathematics for the National Bureau of Standards of the National Research Council (1983–89); Chairman (1985–89)

Panel on Quality Control of Family Assistance Programs of the National Research Council (1986–88)

Graduate Record Examination Board (1984–88)

Visiting Committee to School of Business, University of Miami (1985–89)

Visiting Committee to School of Business, University of Southern California (1989–present)

Board of Mathematical Sciences of the National Research Council (1988–93)

Board of Trustees of Center for Advanced Study in the Behavioral Sciences (1990–present)

## HONORS

Jerry has been honored often for his work. He was a recipient of the Shewhart Medal of the American Society for Quality Control in 1972 and the Cuthbertson Award of Stanford University in 1985 for service to the university. He was selected as a Fellow of the Center for Advanced Study in the Behavioral Sciences in 1985–86 and elected to the National Academy of Engineering in 1987.

## RESEARCH IN STATISTICS

It may be useful to place Jerry's research in statistics in perspective. The period 1945–55 saw considerable activity in the development of sampling inspection plans for military procurement from industry. The Applied Mathematics and Statistics Laboratory at Stanford was funded by the ONR to focus on developing such plans. In addition to Jerry, the other researchers involved were Albert Bowker, Herman Chernoff, Abe Girshick, Henry Goode, Grant Ireson, and George Resnikoff.

A major purpose of the ONR project was to develop a set of lot-by-lot sampling inspection plans based on variables analogous to the plans used when produced items were classified as defective or nondefective. Many alternative sampling inspection plans have been devised and used. Jerry considered a number of these, always motivated by practical considerations arising from an engineering context. In these plans each item in a lot may be classified as defective or nondefective. The problem is to determine the number of items to inspect and classify in order to achieve desired outgoing quality or probabilities of not making a mistake in accepting a bad lot or rejecting a good one. This field has its origin in a paper by H. F. Dodge [1], a statistician at Bell Labs.

### Multistation Lot-by-Lot Sampling Inspection

In practice, it is often the case that an item is defective if it has any of $k$ independent types of defects (e.g., wrong thread size, incorrect size, spotted plating). In that event, inspection for each type of defect frequently occurs at a different station. Jerry's first paper [15], an outgrowth of his dissertation, examines the case in which each of the $k$ stations draws a random sample of $n$ items from a lot; the lot is accepted if the total number of defects from all stations does not exceed a specified acceptance number; and the lot is rejected otherwise. The performance of the plan is evaluated by its operating characteristic (OC) curve, which gives the probability of accepting a lot as a function of the proportion $p$ of the lot that is defective and the probabilities of each type of defect. The main result is that for a common $p$ the OC curve minorizes the OC curve for the case in which all defects arise at a single station and majorizes the OC curve for the case in which the defects are equally likely at each station. The paper tabulates the upper and lower bounds and shows numerically that their difference is small for small $p$, as would typically be so in practice. This is intuitive because the only way the number of defects can differ from the number of defective items is that some item has two or more defects—an event that occurs with small probability.

### Continuous Sampling Inspection

In some industrial contexts items are inspected in a continuous fashion. Dodge proposed the following continuous inspection plan that depends on two positive integers $i$, $k$ as follows: (a) when a defective item is detected, inspect all items until $i$ items in a row are nondefective; (b) then inspect a fraction $1/k$ of the items until a defective item is found, at which point revert to step (a). All defective items found are corrected or replaced by nondefective items. One fundamental, but pessimistic, performance measure of a sampling plan is its average outgoing quality limit (AOQL), i.e., the maximum average outgoing proportion defective over all incoming proportions defective. Dodge calculated the AOQL under the assumption that the production process is in statistical control, i.e., each item has equal probability of being defective and the items are independent. But what if the process is not necessarily in statistical control? To address this issue, it is important to observe first that there are several interpretations of step (b). Jerry considers the case where step (b) has the interpretation that in every block of $k$ items, a single item is inspected at random [16]. He shows that in this event the AOQL is $(k-1)/(k+i)$ and is achieved when all items are nondefective during 100% inspection and all items are defective during partial inspection. His paper with Cyrus Derman and Vernon Johns [28] shows that his AOQL continues to be valid under the alternate interpretation of step (b), in which each item is inspected with probability $1/k$. That paper also examines two variants of Jerry's assumption and develops explicit formulas for the AOQL in those cases.

One drawback of Dodge's sampling plan is that there is an abrupt change between 100% and partial inspection. For large items, such as aircraft engines, this is costly. Thus, with the goals of allowing a smoother change between sampling rates, requiring 100% inspection only when incoming quality is low, and allowing minimal inspection when incoming quality is high, a paper with Herbert Solomon [21] examines a class of multi-level continuous sampling plans. The simplest of these plans depends on three positive integers $i$, $k$, $l$ ($l$ may be infinite) as follows. For each level $j$, $0 \leq j \leq l$, sample at the rate $k^{-j}$. (Note that $j = 0$ corresponds to 100% inspection.) If $i$ consecutive items inspected are nondefective, proceed to level $(j + 1) \wedge l$; if a defective item is found, proceed to level $(j - 1)^+$. As above, all defective items are repaired or replaced with nondefective items. For the case where the process is in statistical control, this paper gives formulas for the average outgoing quality (AOQ) for this plan (and more complex ones) and tabulates the AOQL numerically for $l = 1,2$, and $\infty$. Moreover, the paper shows that both the AOQ and AOQL increase with $l$.

Two other papers [19,25], the second with Albert Bowker, give thorough reviews of continuous sampling inspection plans.

## Sampling Inspection by Variables

In the simplest form of lot-by-lot acceptance sampling, each item of a sample from a lot of manufactured items is classified as defective or nondefective. This method is referred to as "inspection by attributes." When the classification is based on a numerical characteristic of the item, the method is referred to as "inspection by variables." Clearly, the latter procedure uses more information about an item but may be more costly. This method requires an analysis in which characteristics such as means, variances, etc., need to be used.

Jerry's work has delved deeply into the performance of sampling plans for inspection by variables [17,20,23]. His paper with George Resnikoff [20] is the culmination of years of work and is the central paper in this subject. It provides an unbiased estimate of the distribution or survival functions of a normal random variable under several combinations of known and unknown means and variances. This is one of the earliest applications of the Rao-Blackwell theorem for finding unbiased estimates. The paper also uses the range as an estimate of the standard deviation. Perhaps most important are the extensive tables that provide sample size calculations of the OC curves. This work was developed into a Military Standard.

## Estimation of Reliability

Several of Jerry's papers [32,34,35,40,44] relate to the consideration of reliability of a component or system under different sampling procedures and different underlying models. He also provides a general discussion of reliability in the context of an Apollo system for the general reader [42]. This paper is a gem in explaining complicated ideas in lay terms.

The appropriateness of a method for estimating the reliability of a complex system depends in part on the cost of inspection. When the cost of sampling is low, it may suffice to use even inefficient sampling procedures since they may yield accurate estimates by taking large enough samples. However, when the cost of sampling is high (e.g., where tests are destructive), it is important to generate optimal sampling procedures. One paper [32] provides an elaboration of this issue as a consequence of a government requirement that there be an assurance of high reliability where the cost of sampling is high. The proposed solution is based on determining the lower confidence bound for the probability parameter in a binomial distribution.

The reliability of an item is the probability of survival to at least time $t$ and is a function $R(\theta)$ of the parameter $\theta$ of the life distribution. Thus, confidence bounds for the reliability $R(\theta)$ depend on the distribution of $R(\hat{\theta})$, where $\hat{\theta}$ is an estimate of $\theta$. The main result of a paper with Vernon Johns [34] is the determination of an exact lower confidence bound for $R(\theta)$ for the Weibull life distribution, a natural generalization of the exponential life distribution. Extensive tables are provided.

A paper with Sheldon Ross [40] focuses on the reliability of a series system in which each component has an exponential life distribution. The object is to obtain a confidence interval for the system reliability function. One interesting and central result in this development is that the minimum of two independent gamma variables with parameters $(n,\alpha)$ and $(m,\beta)$ is distributed as a mixture of gamma distributions with parameters $(k,\alpha+\beta)$, $k = n \wedge m, \ldots, m+n-1$.

## Regression

A third phase of Jerry's work relates to regression models, once again motivated by industrial problems. The goal is to predict $y$ using $p$ predictor variables $x_1, \ldots, x_p$ in a multiple regression model. Classical theory shows how to fit the regression coefficients using least squares and how to obtain a point estimate or confidence interval for $y$ at a specific point $x = (x_1, \ldots, x_p)$. Jerry shows how to obtain simultaneous confidence bounds for predictions of $y$ at $k$ different values of $x$ [30]. The essence of the problem is that the values are correlated; consequently, individual bounds cannot be combined simply but must be deduced by techniques such as Scheffé's procedure for obtaining simultaneous confidence bounds.

In the paper [30], the number $k$ of predictions is known. Jerry's paper with Rupert Miller [31] assumes instead that $k$ is unknown and possibly large. The solution proposed is that of tolerance intervals. The idea here is to develop an interval $I$ such that $I$ contains $100p\%$ of the distribution (in this case, the normal distribution) centered at the mean of the regression equation at any point $x$ and any $p$ with confidence coefficient $1-\alpha$. Another way of stating this is that a future value of $y$, which depends on $x$, will lie in $I$ with probability at least $p$ with confidence $1-\alpha$. In this formulation the confidence coefficient

$1 - \alpha$ is based on the sample, whereas the probability $p$ refers to the distribution of future observations. This paper examines several alternative procedures for achieving such a tolerance interval.

An interesting inverse of the preceding problem is discussed in a paper with Miller and Martin Hamilton [37]. The paper considers the simple linear regression model $y = \alpha + \beta x + \epsilon$ where one observes $k$ pairs $(x_i, y_i)$, $i = 1, \ldots, k$. In addition, one observes $l$ outcomes $y_1^*, \ldots, y_l^*$ but does not observe the corresponding $x_1^*, \ldots x_l^*$. The problem is to obtain simultaneous confidence intervals for the $x_1^*, \ldots x_l^*$.

### Statistical Tables

Statistical tables were essential components in performing statistical analyses prior to the advent of computers. The tables by Fisher and Yates [2], Pearson and Hartley [6], and others permitted a variety of computations needed to design experiments and analyze data. Because so many applications are based on a comparison of a treatment and a control, Student's $t$ distribution, developed in 1908, is one of the most useful and used distributions. The (null) distribution of the sample standardized mean difference between treatment and control under the hypothesis that the respective true means are equal has been well tabulated. The noncentral distribution, i.e., where the true means differ, is needed in order to determine the power of the test given the sample size and the level of significance, and also to determine the sample size required to achieve a desired level of significance and power. Jerry and George Resnikoff, both motivated by engineering applications, prepared tables of the noncentral $t$ distribution in their 1957 book, *Tables of the Non-Central t-Distribution* [9]. These computations were a tour de force and permitted sample-size determinations for military and industrial applications.

When there are two outcomes, such as defective and nondefective, and sampling is with replacement, the distribution of the number $k$ of defectives in a sample of size $n$ is binomial. In a finite population with $K$ defectives and $N$ nondefectives, and sampling is without replacement, the distribution of the number $k$ of defectives in a sample of size $n$ is hypergeometric. Because this distribution is difficult to compute when $K$ and $N$ are large, Jerry and Donald Owen prepared tables of the hypergeometric distribution in their 1961 book, *Tables of the Hypergeometric Probability Distribution* [10]. These tables permit the evaluation of sampling plans without replacement. Whereas some tables can now be reproduced more readily by modern computers, that is less so for the hypergeometric distribution, and these tables remain useful to this day.

## RESEARCH IN OPERATIONS RESEARCH

Jerry's research in operations research began in the mid-1950s. He found a kindred spirit in another statistician, Cyrus Derman, in Industrial Engineering at

Columbia University. Jerry visited Columbia in the autumn of 1957 and excited the students there (including Arthur Veinott) about the work in dynamic programming and its applications to inventory problems that was being done at Stanford by Arrow, Karlin, and Scarf.

### LIFO and FIFO Issuing Policies

While at Columbia, Jerry became interested in Derman and Morton Klein's work on the optimality of LIFO (last in, first out) and FIFO (first in, first out) issuing policies for items whose field life or value depends on their age at issue. This led to Jerry's first paper in operations research [26] (partly in collaboration with Aryeh Dvoretzky), which showed that if the *life* of an item, i.e., the sum of the item's age at issue and its subsequent field life, rises with its age at issue and if LIFO (respectively, FIFO) maximizes the field life of two items, then that policy does likewise for an arbitrary number of items. The paper also showed that if the life of an item rises with its age at issue and its field life is monotone and concave in its age at issue, then FIFO maximizes the total field life of the items. The special case in which the field-life function is decreasing and concave is natural in practice (at least in the relevant interval of ages before the field life becomes negative) and justifies the use of FIFO in many circumstances.

### Collaboration with Derman and Ross

One of Jerry's Ph.D. students, Sheldon Ross, took his degree in Statistics in 1967 and has since been at Berkeley in Industrial Engineering and Operations Research. Jerry shared with Derman and Ross a common background in statistics and an interest in the development of natural stochastic optimization models in operations research. Their interests and outlook led them to begin a fruitful collaboration that continued for two decades and produced some 15 papers. Sheldon Ross described how their collaboration started in a recent communication to us:

> I was working with Jerry on his grant in London in the spring of 1970 when Cy Derman passed through, so we took an afternoon off to show him the sights. While riding around in a taxi we discussed a problem that led to the paper [41] and the Derman-Lieberman-Ross team was born.

An association like this does not endure for so long without a close relationship among the participants. In a recent communication to us, Cyrus Derman put it this way:

> We enjoyed a long collaboration and were always pleased to find that at the end of the summer's work we had produced a publishable piece of research. A number of times the results were interesting enough to stimulate other researchers and Ph.D. dissertations. But what I value most from our collaboration was the light-hearted manner in which we worked and the lasting friendship.

The main theme of their papers is optimal sequential or nonsequential decisions in the presence of uncertainty for particular models of potential practical interest, usually relating to optimal design or maintenance of reliable systems. The goal is frequently to characterize the form of an optimal policy and to develop algorithms for finding such a policy. Existing general theories are sometimes applied to help solve these problems. But more often the authors prefer to examine each problem in its own terms and develop their results afresh, not infrequently in clever ways. This approach has the added benefit of suggesting interesting questions about the generality of the ideas for future exploration. We discuss several of their papers here.

### Joint Stocking and Replacement Decisions

A paper in 1967 [36] (in which Ross assisted, but did not co-author) concerns joint stocking and replacement decisions—one of the first to do this—for a part whose performance level is random and whose failure rate is independent of the performance level. The performance level of a part is observed when it is placed in service and remains fixed thereafter. In each period a working part is either retained in service or replaced from stock if available. A failed part is replaced from stock if available. When the parts inventory is exhausted, an order is placed and delivered at the beginning of the next period. A part that is retained in service for a period fails with given performance-dependent probability during the period. During each period a cost is incurred. When there is a positive inventory, that cost depends on the inventory and the performance level of the part in service, and falls as the performance level rises. The goal is to minimize the long-run expected average cost per period. The optimal restocking policy is to order a common amount each time the stock of parts runs out. The optimal replacement policy entails retaining a part if its performance level is at least an acceptable inventory-dependent performance level and replacing the part otherwise. If also, the one-period cost with a positive inventory is additive and rises with the inventory of parts, then the optimal acceptable performance level rises as the parts inventory rises. A version of the policy improvement method for solving the problem is developed that exploits these and other structural results to improve the computational efficiency of the method.

### Multiple Assignment Problem

Several of the Derman-Lieberman-Ross papers apply and extend a result of Lorentz [5] on the *multiple assignment problem* in interesting ways. Because Lorentz's result plays a central role, we review it briefly here in an updated setting. Suppose $r$ is a real-valued function of $m$ variables. Also suppose $P$ is an $n \times m$ matrix whose $ij$th element is $p_i^j$ and whose $i$th row is $p_i$. Put

$$R(P) = \sum_{i=1}^{n} r(p_i). \qquad (1)$$

The problem is to chose an $n \times m$ matrix $Q$ that minimizes $R(Q)$ among those for which each column of $Q$ is a (possibly different) permutation of the elements of the corresponding column of $P$. Lorentz considers the case in which $r$ is *lattice superadditive*. For purposes of discussion, it suffices to take this to mean that the mixed second differences of $r$ with respect to each pair of variables is nonnegative. If $r$ is twice continuously differentiable, this is equivalent to saying that the mixed second-order partial derivatives of $r$ are nonnegative. Lorentz shows that if $r$ is lattice superadditive, then one optimal choice of $Q$ is *monotone*, i.e., the $i$th row of $Q$ is increasing in $i$. As an example, notice on letting $\pi = (\pi^1, \ldots, \pi^m)$ that $r$ is lattice superadditive if

$$r(\pi) = \prod_{j=1}^{m} \pi^j \tag{2}$$

and either $m = 2$ or $m > 2$ and $\pi \geq 0$. When $m = 2$ and Eq. (2) holds, Lorentz's result specializes to a result in Hardy, Littlewood, and Pólya [3, p. 261].

### Stochastic Sequential Assignment Problem

Jerry, Derman, and Ross [41] provide an interesting generalization of Hardy, Littlewood, and Pólya's and of Lorentz's results for the case $m = 2$. It is easiest to explain the problem by interpreting the elements in the two columns of the $n \times 2$ matrix $P$, respectively, as the skills of $n$ individuals and the profits of $n$ tasks. Then, $r(s, p)$ is the reward an individual of skill $s$ earns when executing a task whose profit is $p$. In this setting the problem is to assign the $n$ tasks to the $n$ individuals in order to maximize the sum of the rewards the individuals earn. The $n$ tasks and their profits, as well as the $n$ individuals and their skills, are known in advance. The generalization of the problem that they consider assumes instead that one new task arrives in each of $n$ periods and the profits of the tasks are random variables with the profit of a task being first revealed when it arrives [41]. Label the individuals $1, \ldots, n$ in order of increasing skill. Their paper [41] shows that if $r$ is lattice superadditive and if the profits of the $n$ tasks are independent random variables, then the optimal individual to assign to the first task is an increasing function of the profit of that task. In the special case where the task profits are period-dependent constants, this amounts to a reformulation of Lorentz's result. Unfortunately, the computational effort to calculate the optimal initial assignment in the stochastic case by dynamic-programming-style backward induction increases exponentially with $n$ because the optimal initial assignment depends on the skills of the $n$ individuals. However, when Eq. (2) holds, the paper [41] finds a striking improvement of the preceding result. This case is natural in a number of applications, e.g., where the "skill" of an individual is the probability that the individual can successfully complete a task independently of its profit. In that event the goal is to assign tasks to individuals to maximize the expected value of total profits earned. The improvement in the result for this case is that the optimal initial assignment depends on the number of individuals, but not their skills, and the maximum expected profit is linear in those skills. This permits the computational effort to find an optimal sequential assignment policy to be reduced to $O(n^2)$.

## Optimal Assignment of Components of Several Types to Systems

The series of Derman-Lieberman-Ross papers [43,46,49,52] examines various problems of assigning components of several types to systems to maximize the expected number or stochastically maximize the number of working systems. The first paper [43] applies and extends Lorentz's [5] and Hardy et al.'s [3] results in nice ways. The paper begins by considering the problem of assembling $m$ types of components into $n$ systems where there are $n$ components of each type and a system requires one of each type. The $i$th component of type $j$ has a known numerical characteristic $p_i^j$, e.g., weight, tensile strength, conductivity, reliability, quality, etc. A system works if the random required characteristic of each of its components does not exceed the actual characteristic of the component. Denote by $r$ the joint distribution of the random required characteristics of its components. As earlier, let $p_i$ be the $i$th row of the matrix $(p_i^j)$. Then $r(p_i)$ is the reliability of a system formed from the $i$th component of each type and Eq. (1) gives the expected number of working systems. The problem is to assign components to systems to maximize the expected number of working systems. This is an instance of Lorentz's problem given already. Since $r$ is the joint distribution of the random required characteristics, $r$ is lattice superadditive. Thus, by Lorentz's result, the assignment that maximizes the expected number of working systems entails assembling a system whose components of each type have the lowest characteristic, then one whose components of each type have the next lowest characteristic, etc. When the reliability of a system is at least one-half under all assignments, as would often be so, the paper shows that this solution also minimizes the variance of the number of working systems.

## Series, Parallel, and $k$-of-$n$ Systems

Systems typically combine components in various ways. Two of the simplest types of systems are series and parallel. In *series* (respectively, *parallel*) systems, every (respectively, one) component must work for the system to work. Series systems arise frequently in practice. Parallel systems arise in various ways—notably where redundancy is used to increase reliability, e.g., as in fault-tolerant computing or in achieving high reliability with inexpensive components. A useful generalization of a series and of a parallel system is a *k-of-n system*, which is a system in which at least $k$ of $n$ components must work for the system to work. For example, a four-engine plane in which two engines must work for the plane to fly is a 2-of-4 system. Of course, series systems are $n$-of-$n$ systems and parallel systems are 1-of-$n$ systems.

### Optimal Assignment of Components to Series and Parallel Systems

Jerry, Derman, and Ross [43] consider a second component-assignment problem – this time for series systems. In this problem the $i$th component of type $j$ has known reliability $p_i^j$, the components are independent and $r(p_i)$ is the reliability of a system consisting of the $i$th component of each type where, as above, $p_i$ is the $i$th row of the matrix $(p_i^j)$. Then, Eq. (2) holds, and Eq. (1) gives the expected number of working systems. As already mentioned, this implies that $r$ is lattice superadditive. Thus, if the goal is to maximize the expected number of working systems, it follows from Lorentz's result that it is optimal to assemble a system whose components of each type have lowest reliability, then one whose components of each type have next lowest reliability, and so on. Their paper [43] establishes the stronger result that this procedure also stochastically maximizes the number of working systems. Hence, if a $k$-of-$n$ master system is formed from the $n$ systems, the procedure also maximizes the reliability of the master system for every $k$. Also, because $P$ is monotone and $r$ is increasing, the reliability $r(p_i)$ of the $i$th system is increasing in $i$.

Another paper [46] first discusses a variant of the preceding problem for parallel systems. Suppose that there are $n$ independent components with $p_i$ here being the reliability of the $i$th component. The problem is to assign the $n$ component to up to $m$ parallel systems to maximize the expected number of working systems. An optimal assignment equalizes system reliabilities if that is possible. In the contrary event, the paper gives bounds on the maximum expected number of working systems. Observe that an optimal assignment in parallel systems is opposite to that in serial systems. Whereas the former entails minimizing the difference between the largest and smallest of the system reliabilities, the latter entails maximizing that difference.

### Optimal Allocation of Component Reliabilities in $k$-of-$n$ Systems

Next, the paper [46] considers the problem of choosing the component reliabilities $p_i$ to maximize the reliability of a $k$-of-$n$ system subject to the constraints that the component reliabilities lie in the interval $[0,1]$ and that their sum equals a fixed "reliability budget." It is optimal to allocate the reliability budget equally among all components in series systems and, when the budget is at most one, to allocate the entire reliability budget to a single component in parallel systems. More generally, in $k$-of-$n$ systems, it is optimal to allocate the entire reliability budget equally among $m$ of the components for some $m \geq k$. The paper also considers the generalization of these results to the problem of maximizing the expected number of $k$-of-$n$ systems that work where $nM$ components are manufactured and assembled into $M$ systems each having $n$ components, and each component in a system is assumed to have the same reliability. Subject to this constraint, the optimal allocation depends on two numbers $p$ and $q$ with each component reliability equalling 0, $p$, or $q$.

### Optimal Allocation of a Budget to Components in a *k*-of-*n* System

In practice, it is generally more reasonable to consider allocating a fixed budget of funds rather than of reliabilities. For this reason, another paper with Derman and Ross [49] supposes that the probability $p(x)$ of a component working depends on the amount $x$ spent on the component and the goal is to maximize the reliability of a $k$-of-$n$ system. The paper considers both the sequential and nonsequential cases. In the sequential case, one spends $x_1$ on the first component and observes the outcome, spends $x_2$ on the second component and observes the outcome, etc., until one has $k$ working components or tries $n$ times, whichever comes first. For both the sequential and nonsequential cases, the paper shows that it is optimal to allocate the budget equally among the components if $\log(1-p(x))$ is convex, and when $k=1$, to a single component if $\log(1-p(x))$ is concave. Incidentally, the function $\log(1-p(x))$ is convex (respectively, concave) if and only if the incremental conditional success rate, i.e., hazard rate, falls (respectively, rises) with the amount $x$ spent on a component. The paper also reexamines the case where $p(x)=x$, i.e., where the problem is to allocate reliabilities to components. The solution is given earlier for the nonsequential case. For the sequential case, the paper develops the solution when $k=2$.

### Optimal Expenditures on Components to Build a *k*-of-*n* System

The paper [49] suggests an alternate formulation of the problem that is considerably simpler, viz., sequentially minimizing the total expenditures to build a $k$-of-$n$ system. In this event if one spends $x$ each time one tries to build a component, the expected number of tries to build a working component is $1/p(x)$. Thus, it suffices to choose $x$ to minimize the expected cost $x/p(x)$ of building a single working component and repeat this $k$ times to build the $k$ components. This formulation allows an unlimited number of tries. What if that is not possible?

Again with Derman and Ross [52], Jerry considers a variant of the problem in which the goal is to sequentially minimize the total expenditures to build $k$ components in at most $n$ tries where there is a penalty cost $c(i)$ when $i \geq 0$ of the components remain to be built and no tries remain. In that event, the main result is that if $c$ is convex increasing on the nonnegative integers and vanishes at origin, the optimal amount to spend on the first component rises with $k$ and falls with $n$. The paper also shows that when $c(i)=ci$, the optimal amount to spend on the first component rises as $c$ rises.

### Optimal Choice of Replacement Component Types

Another paper with Derman and Ross [53] considers the problem of maintaining a system over a finite horizon with minimum expected cost. The system (e.g., a car) has a component (e.g., a battery) whose life distribution is exponential and that must be replaced when it fails. Several types of replacement components with differing costs and failure rates are available. This is a common situation arising in practice where an important question is whether or not spending more to buy a more reliable component type is justified. The answer depends on the horizon length. To understand the issue at hand, consider the optimal choice in two extreme cases. When a short enough time remains, using the cheapest component is best because it is unlikely to require replacement; but when a long enough time remains, using the component with lowest cost per unit time is best because it will be replaced often. But what is optimal between these extreme cases? The paper [53] and a companion note of Donald Smith [7] together show that it is not optimal to use a component type that has both higher cost and higher cost per unit time than those of another component type. Thus, after deleting these (nonoptimal) component types, it is possible to label the component types $1, \ldots, n$ so that both the cost rises and the cost per unit time falls as the component type rises. The paper [53] also shows that the optimal replacement component type to use when a failure occurs with time $t$ remaining rises from one to $n$ as $t$ traverses the positive real line. This nice solution bridges the gap between the two extreme cases discussed earlier. Moreover, this paper shows how to calculate the optimal policy by induction on $n$.

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